Use of CCDs for Fringe Tracking

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P.1. SUMMARY

CCD detectors have several attributes which are desired for low light level imaging applications. These include high quantum efficiency, fairly uniform response across the image area, 100% fill factor, and very low dark current (with cooling). The drawbacks to CCD imaging are that these devices are inherently framing devices and there is a noise penalty associated with each pixel read.

In this appendix I show that CCD technology has matured so that a bare CCD camera is the detector of choice for the fringe tracking task, given that we can obtain a CCD with readout noise of 2 electrons noise or less at adequate frame rates.

P.2. INTRODUCTION

Using the channel spectrum to phase the CHARA array has been proposed by Theo ten Brummelaar; this approach to phasing separate telescopes is presently being pursued by the IOTA array on Mt. Hopkins and the SUSI array in Australia.

Wes Traub of the Smithsonian Astrophysical Observatory described their implementation of this scheme. I discuss operational aspects of this approach in a companion document (see Appendix J). In this appendix, I will not discuss the specifics of the channel spectrum except to state that it may provide us with a one-dimensional fringe in wave number space. We also assume that we can use a CCD array to read out pixels that are equally spaced in wavenumber domain. We will need to do some on-chip binning to achieve the equal spacing, but we can assume that the binning process is noise-free and that the readout noise is the same per increment in wavenumber space.

For this discussion, let us assume that we are trying to unambiguously detect and measure the phase and frequency of a one-dimensional fringe on a CCD chip. The intensity signal, $I(\sigma)$, of the channel spectrum fringe pattern can be written as

$$I(\sigma) = S(\sigma)QE(\sigma)(1 + V(\sigma)cos(2\pi\sigma\Delta x(\sigma) + \alpha(\sigma))) \tag{P.1}$$

where σ is the wavenumber ($= 1/\lambda$) expressed in units of μm^{-1}, $S(\sigma)$ is the spectral distribution of the object, $QE(\sigma)$ is the quantum efficiency of the optics-detector system, $V(\sigma)$ is the visibility of the object, $\Delta x(\sigma)$ is the pathlength difference of the two interfering beams, and $\alpha(\sigma)$ is a phase offset that will be non-zero for objects other than a point source.

The channel spectrum is ambiguous as to which beam is "ahead" of the other, but that is not important for our present discussion.

P.3. CCD NOISE BEHAVIOR

The readout noise of a CCD chip is characterized by a Gaussian distribution about a mean of zero. The noise is uncorrelated between pixels. The noise spectrum can be modeled as a combination of $1/f$ and white noise processes. For readout rates higher than the knee of the noise spectrum (typically $\sim$50 kHz), the readout noise generally increases as the square root of the readout rate. The best noise performance yet reported by a scientific grade device is 1.4 electrons (e^-) (rms) at 50 KHz and 5–6 e^- at 1 MHz. On a rare night on Kitt Peak, we once attained 6.2 e^- at 1 MHz with the GTRI CCD imaging system; however, more typically we get 7 e^- at that rate. It is physically possible to achieve single electron noise at MHz readout rates and the Japanese have done it for a non-scientific interline transfer CCD chip. I am aware of some new scientific grade CCD chip designs that should soon attain 3 e^- noise at 1 MHz, thus it is reasonable to project for CHARA a readout noise of 3 e^- at 1 MHz.

I suggest that we read out 128 pixels per channel spectrum. For 7 spectra on a CCD chip, we will need to read out 896 pixels per frame. With multiple readout ports and a properly designed chip, it will not be difficult to achieve 32 electron noise (rms) with readout time of about 10 msec. We have determined that we could use our present CCD camera and achieve a readout time of 3.2 msec. However, the CCD chip in that camera, a 420×420 pixel frame transfer device with a single readout port, is a rather poor design for this application. A proposed CCD chip design for the tracking camera will be specified later.

Note that implementation of on-chip pixel binning, to collapse spectra into one-dimension and achieve equally spaced samples in wavenumber space, will make the effective pixel size be rather large. This can lead to substantial dark current noise, unless the chip is adequately cooled. We will need to use liquid nitrogen cooling to reduce the dark current to negligible levels.

P.4. SIGNAL-TO-NOISE RATIO (SNR) OF FRINGE DATA (PHOTON NOISE ONLY)

Given the task of detecting and measuring the amplitude and phase of a 1-D fringe, we can turn to the work of several different investigators. Perhaps the first analysis in this vein was done by Walkup & Goodman (1973). Other related papers are those by Goodman & Belsher (1976), Dainty & Greenaway (1979), Roddier (1986), Buscher (1988), and Beletic & Goody (1992). All of these papers build on the early work of Walkup & Goodman which discussed the measurement of a fringe across the pixels of a 1-D photon-counting camera.

(Note that after re-examination of the SNR equations, I think that the Walkup & Goodman paper neglected the effect of transfer function, which for a single fringe has the effect of reducing the SNR by a factor of 2. The following equations have this modification, which must be re-examined and verified.)

Their results, which led to the SNR analysis of speckle interferometry and speckle imaging, are as follows. If we denote the number of detected photons as N_{ph}, and the fringe visibility as V_s and we make the assumption that background counts due to dark current and unwanted light is negligible, the SNR of the fringe measurement is given by the quantity γ,

$$\gamma = \frac{V_s}{2\sqrt{2}}\sqrt{N_{ph}} \qquad \text{(P.2)}$$

For values of γ greater than $\sqrt{2}$, a very good approximation to the variance of the phase measurement is given by,

$$\left\langle \Delta\phi^2 \right\rangle \approx \frac{1}{\gamma^2} = \frac{8}{V_s^2 N_{ph}} \tag{P.3}$$

Rearranging, we have an expression which we can use to estimate the number of photons that we will need to detect in order to achieve a given level of performance,

$$N_{ph} \ = \ \frac{8}{V_s^2 \left\langle \Delta\phi^2 \right\rangle} \tag{P.4}$$

P.5. EFFECT OF READOUT NOISE ON FRINGE MEASUREMENTS

The present plan is to measure the fringe of the channel spectrum by taking the Fourier transform of the spectrum to detect a peak and then move baselines to position the peak at a desired frequency.

Although the SNR of fringe detection in the case of readout noise has not been formally derived, the situation appears to the be same as that of speckle imaging, for which Zadnik (1993) has recently completed a thorough analysis. In speckle imaging, the situation is more complex, since the fringe patterns are distorted by the atmosphere and the photon noise combined with the varying intensity pattern produces a *compound Poisson process.*

For this first order analysis of the effect of readout noise on the channel spectrum, we have a simpler situation. We need to evaluate the effects of photon noise and readout noise on the measurement of a fringe across a one-dimensional detector. In any low light level imaging case, there is always noise present at a frequency due to signal at twice that frequency; this is the interesting "half-frequency" phenomenom discussed by Goodman (1985). For this analysis, we assume that there is no energy at twice the frequency of interest and thus ignore those terms and other terms that are insignificant, and we find that the SNR of a signal a frequency ω is given by,

$$\mathrm{SNR}(\omega) = \frac{N^2|\hat{T}(\omega)|^2|\hat{O}(\omega)|^2}{\sqrt{N^2 + 2N^3|\hat{T}(\omega)|^2|\hat{O}(\omega)|^2 + n_{pix}^2\sigma_{CCD}^4 + 2n_{pix}\sigma_{CCD}^2(N^2|\hat{T}(\omega)|^2|\hat{O}(\omega)|^2 + N)}} \tag{P.5}$$

where N is the number of detected photons, $O(\omega)$ is the complex object spectrum at frequency ω, $T(\omega)$ is the atmosphere-telescope transfer function, $n_{pix} =$ is the number of pixels read out of the CCD and $\sigma =$ is the CCD readout noise (rms electrons). The tophat symbol $(\hat{})$ denotes the transfer function and object spectrum have been normalized, i.e. divided by the magnitude of the DC term. Normalization limits magnitude of the spectra of all real telescopes and astronomical objects to lie within the range of 0 to 1. The magnitude of the complex spectrum, $|\hat{O}(\omega)|$, is the *visibility*, $V(\omega)$, at frequency ω.

For the case of a single fringe across a 1-D detector, $|\hat{T}(\omega)|$ equals 1/2 at spatial frequency we are measuring and the above equation reduces to,

$$\mathrm{SNR}(\omega) = \frac{\frac{1}{4}N^2V^2(\omega)}{\sqrt{N^2 + \frac{1}{2}N^3V^2(\omega) + n_{pix}^2\sigma_{CCD}^4 + 2n_{pix}\sigma_{CCD}^2(\frac{1}{4}N^2V^2(\omega) + N)}} \tag{P.6}$$

Note that the effect of the readout noise is similar to that from thermal dark current (which we assume is negligible due to cooling), in that it is the variance of the noise "signal" that is most important. Thus, although most researchers quote their readout noise by stating the rms fluctuation of the dark signal, the effect of the noise in the spatial frequency domain is proportional to the variance of that fluctuation. Thus, although it is a zero mean Gaussian process, readout noise can be thought of as originating from a "noise signal" of σ^2 readout electrons per pixel.

For fringe detection to have adequate SNR in a photon counting camera, the signal dependent noise (second term in the denominator) must dominate the white photon noise (first term). As can be seen from the equation, the readout noise of a CCD detector starts to become negligible when the average number of photons per pixel, N/n_{pix}, is less than σ^2. Thus, it is advantageous to read out the minimum number of pixels and of course, one wishes to obtain the lowest readout noise detector available.

Figures P.1 – P.4 show the SNR that is obtained as a function of number of incident photons for a number of different detectors. The two darkest curves represent the performance of perfect photon counting cameras at 10% and 80% quantum efficiency. The very best photon counting camera may be able to achieve the 10% quantum efficiency level and so that should represent the level of performance that can be obtained with those devices. The 80% "perfect camera" curve is a fair representation of the best that we could ever hope to achieve with a ultra-low noise CCD camera, or avalanche photodiode array (when they arrive in the future). The curves in between represent the level of performance that can be attained with different quality CCD cameras, all of which are assumed to have 80% quantum efficiency.

A horizontal line is drawn on the graphs for SNR = 0.7, the SNR required for fringe tracking when we can average the power spectra of 50 frames to follow the signal, as discussed in Appendix J.

As expected, the SNR is a strong function of visibility, with Figures P.1 – P.2 showing the performance for a visibility of 0.312 ($|V|^2 = 0.1$) and Figures P.3 – P.4 showing the performance for visibility of 0.8 ($|V|^2 = 0.64$). The curves also show that the crossover point at which one would switch from a photon counting camera to a CCD is primarily a function of incident light level and is not very dependent upon the visibility.

P.6. CONCLUSIONS

A preliminary conclusion that I draw from these plots is that we should read out the minimum number of pixels for each channel spectrum. We can use 128 pixels per channel spectrum with the option of reducing that number to 64 for the faintest objects. The constraint that we have with a smaller number of pixels is the inability to detect a large number of fringes across the spectrum and problems with aliasing of higher spatial frequencies. A smaller number of pixels will make it harder to watch the channel spectrum to see two apertures come into phase.

We can also conclude that if the channel spectrum is complicated due to object structure

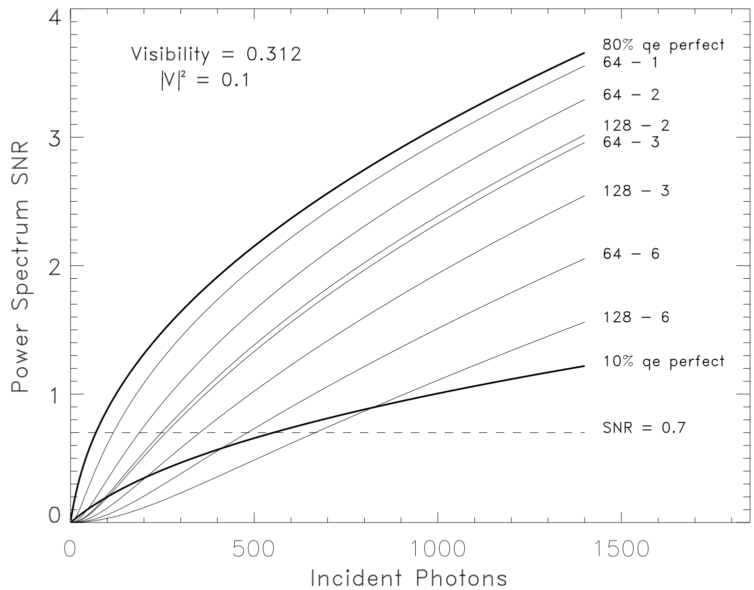

FIGURE P.1. Fringe detection signal-to-noise ratios, for a visibility of 0.312. Dark lines indicate performances of "perfect" photon counting cameras at 10% and 80% quantum efficiency. Other curves represent levels of performance that can be attained with different quality CCD cameras, all of which are assumed to have 80% quantum efficiency; to the right of each curve are number of pixels per channel spectrum and readout noise. The horizontal dashed line at a SNR = 0.7 indicates the signal-to-noise ratio required for fringe tracking.

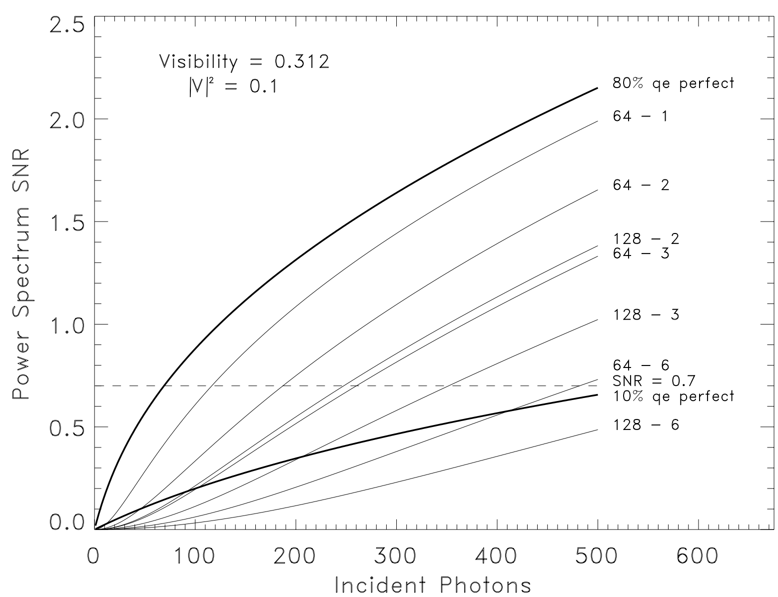

FIGURE P.2. Blow up of Figure P.1 for a smaller number of incident photons.

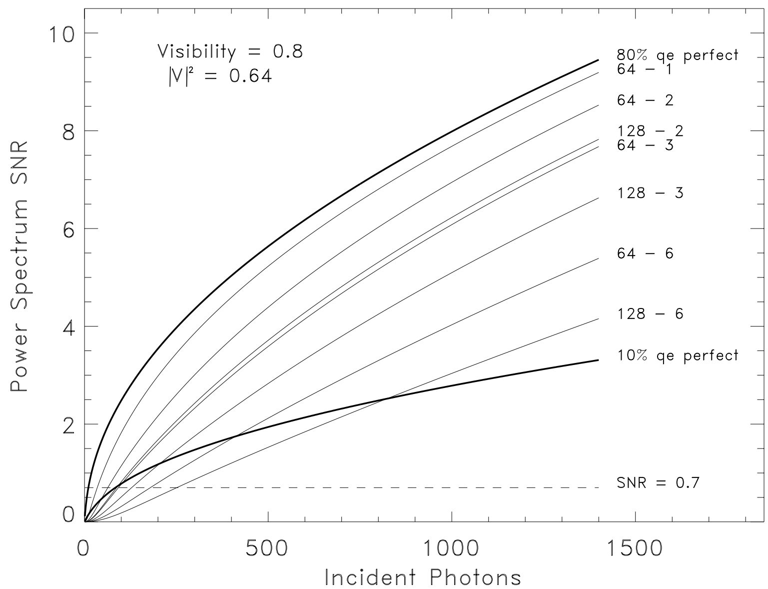

FIGURE P.3. Fringe detection signal-to-noise ratios, for a visibility of 0.8.

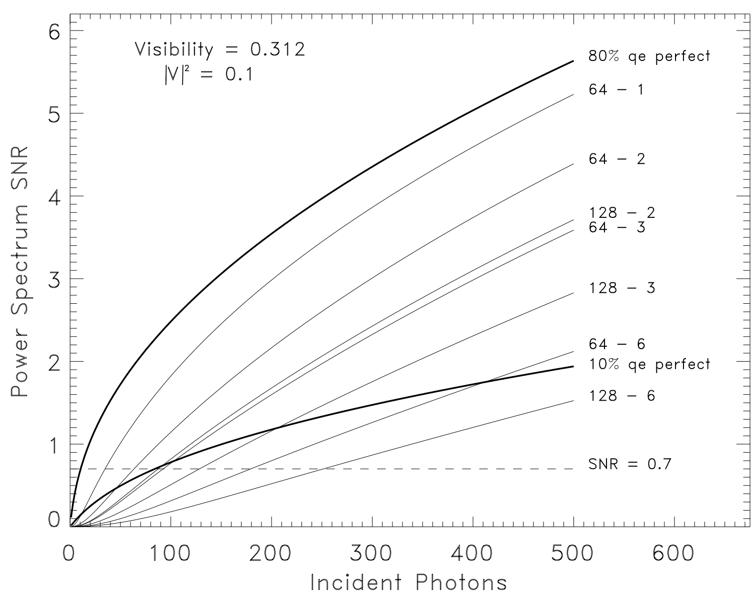

FIGURE P.4. Blow up of Figure P.3 for a smaller number of incident photons.

so that we require more than 1000 incident photons to obtain an adequate SNR, then the CCD is easily the detector of choice. In fact, there may be very few complicated objects that are bright enough to be tracked with a photon counting camera.

Also, it appears that we need to strive for readout noise of 2 electrons (rms) or less. We are involved with groups that are leading the effect in this direction and this analysis should provide more emphasis to keeping those contacts current. These groups include MIT Lincoln Laboratory which is pursuing improved FET designs and JPL which is pushing the skipper amplifier design. I project that 2 electron read noise will be attained at adequate read rates within 1 year by one of these approaches.

One should also keep in mind that as read noise is reduced, the thermal noise becomes more important, so we will probably need to go to liquid nitrogen cooling for the fringe tracking camera at the lowest readout noise levels.

P.7. REFERENCES

Beletic, J. W. & R.M. Goody, R. M., 1992, "Recovery of planetary images by speckle imaging", *Appl. Optics*, **31**, 6909

Buscher, D., 1988, "Getting the most out of C.O.A.S.T.", Ph.D. thesis, Cambridge University

Dainty, J. C. & Greenaway, A. H., 1979, "Estimation of spatial power spectra in speckle interferometry", *JOSA*, **69**, 786

Goodman, J. W., 1985, *Statistical Optics*, (Wiley, New York)

Goodman, J. W. & J. F. Belsher,J. F., 1976, "Fundamental limitations in linear invariant restoration of atmospherically degraded images," SPIE Proc., **75**, 141

Roddier, F., 1986, "Pupil plane versus image plane in Michelson stellar interferometry", *JOSA-A*, **3**, 2160

Walkup, J. F. & Goodman, J. W., 1973, "Limitations of fringe-parameter estimation at low light levels", *JOSA*, **63**, 399

Zadnik, J. A., 1993, "The use of charge coupled devices in astronomical speckle imaging", Ph.D. thesis, Georgia Institute of Technology, (in review)